Complex Permittivity Determination from Measured Scattering Parameters of TEM Waveguides

Complex Permittivity Determination from Measured Scattering Parameters of TEM Waveguide

P. Savi,* U. Niyazov,† I.A. Maio‡

Abstract — **This paper addresses the de-embedding of the propagation function of waveguides from the scattering responses of setups composed of TEM waveguides terminated by launchers that introduce generic discontinuities. The de-embedding is aimed at estimating the permittivity of dielectric samples from the scattering responses of waveguides including the samples. The de-embedding is based on the double-delay method, that is applied to setups involving different launchers.**

1 INTRODUCTION

An important method to estimate the permittivity of dielectric materials is based on the measurement of the scattering parameters of a TEM (Transverse Electro-Magnetic) waveguide filled by the dielectric to be characterized and on the inversion of the scattering response for the unknown permittivity. This approach is exploited in many applications where the estimation of the permittivity over wide frequency bands is required, as in the characterization of dielectric materials for electronics packaging and in the measurement of the permittivity of soils in soil science. In a uniform TEM waveguide the relation between the permittivity of the filling dielectric and the propagation function is simple, and the resulting inversion problem is readily solved. In order to connect a uniform TEM waveguide to a Network Analyzer for the measurement of the scattering parameters, however, the waveguide must be completed by suitable launchers at its ends. Depending on the specific application, the launchers can introduce a significant discontinuity, and can lead to a transmission response of the waveguide plus launchers quite different from the propagation function of the waveguide alone. The problem then becomes how to eliminate the effects of launchers from the measured scattering responses of the waveguide and its launchers (de-embedding), obtaining the transmission response of the waveguide alone. For this problem, several method have been developed, *e.g.,* see [1], [2] and [3]. The double-delay method of [2], in particular, seems well suited to the de-embedding of the propagation function of a waveguide terminated by arbitrary launchers.

In this paper, we apply the double-delay method to estimate the complex dielectric permittivity from the measured scattering responses of TEM waveguides. The double-delay method of [2] is based on the scattering responses of a pair of test structures composed of a segment of the waveguide being characterized and its launchers. The two test structures must differ for the length of the waveguide segment only. Furthermore, the shortest waveguide segment must be long enough to guarantee that a pure TEM propagation takes place for a part of the segment. Two coaxial probes of different lengths have been used in this study to measure the scattering response and the ability of the double-delay method to de-embed the effect of the launchers and to correctly estimate the complex dielectric permittivity has been verified on reference liquids and dielectric samples.

2 ANALYSIS

The double-delay method of [2] that we use in this study is based on the scattering responses of a pair of test structures composed of a segment of the waveguide being characterized and its launchers. The two test structures must differ for the length of the waveguide segment only. Furthermore, the shortest waveguide seg-

*Dip. Elettronica, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy, tel.: +39 011 5644074, e-mail:patrizia.savi@polito.it

†ibidem, umid.niyazovsokhibovich@studenti.polito.it

‡ibidem, maio@polito.it

ment must be long enough to guarantee that a pure TEM propagation takes place for a part of the segment. In contrast, the left and right launcher can be different, *i.e.,* no longitudinal symmetry is required.

Let $\ell_a < \ell_b$ be the lengths of the two waveguide segments, and $\mathbf{S}_{ta}$ and $\mathbf{S}_{tb}$, the transmission scattering matrices of the setup with the ℓ_a and ℓ_b segment, respectively, then

$$
\begin{aligned}
\mathbf{S}_{ta} &= \mathbf{X}_1 \left[\begin{array}{cc} \exp\{-\gamma(s)\ell_a\} & 0 \\ 0 & \exp\{+\gamma(s)\ell_a\} \end{array} \right] \mathbf{X}_2 \\
\mathbf{S}_{tb} &= \mathbf{X}_1 \left[\begin{array}{cc} \exp\{-\gamma(s)\ell_b\} & 0 \\ 0 & \exp\{+\gamma(s)\ell_b\} \end{array} \right] \mathbf{X}_2
\end{aligned} \tag{1}
$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are the transmission scattering matrices of the left and right launchers, respectively, s is the Laplace variable and $\exp\{+\gamma(s)z\}$ is the propagation function of the waveguide for a propagation distance z, $\gamma(s)$ being the propagation constant. In the above equation, the diagonal matrices represent the transmission scattering matrices of the two waveguide segments, which implies that the reference impedances for the waveguide ports coincide with the waveguide characteristic impedance. The matrices $\mathbf{S}_{ta}$ and $\mathbf{S}_{tb}$ can be obtained from the scattering matrices of the two test structures, whereas $\mathbf{X}_1$, $\mathbf{X}_2$ and the propagation function are the unknowns of the problem. Of course, the reference impedances of the wave variables at the launcher ports are the VNA calibration impedance and the waveguide characteristic impedance. The latter, therefore, is a supplemental unknown of the problem.

When $\mathbf{X}_2$ is computed from the first equation of (1) and replaced into the second one, the following eigenvalue equation for $\mathbf{X}_1$ arises

$$
[\mathbf{S}_{tb}\mathbf{S}_{ta}^{-1}]\,\mathbf{X}_1 = \mathbf{X}_1 \left[\begin{array}{cc} \lambda_1 & 0 \\ 0 & \lambda_2 \end{array} \right] \tag{2}
$$

where $\lambda_1 = \exp\{-\gamma(s)(\ell_b - \ell_a)\}$ and $\lambda_2 = \exp\{+\gamma(s)(\ell_b - \ell_a)\}$. An analogous equation holds for $\mathbf{X}_2$.

For every frequency value, the measured scattering matrices yields six independent parameters (three for each test structure), whereas the unknowns of the problem are the six scattering parameters of the launchers, and the propagation function and the characteristic impedance of the waveguide. The measured data, therefore, do not allow a complete deembedding of the waveguide responses (*e.g.,* see also [2, 5]). For the inversion problem at hand, however, the eigenvalues of (2) are the samples of the propagation function and, provided the relation between $\gamma(s)$ and the dielectric permittivity is known, they allow to compute the unknown permittivity.

In order to compare the estimation of the propagation function via the double-delay method with the Nicolson-Ross method [1], it is expedient to formulate the latter in terms of transmission scattering matrices. The Nicolson-Ross method uses the scattering responses of one setup only (*e.g.,* the one with the ℓ_a long waveguide), that must be symmetric. The transmission scattering matrix of the measured responses, therefore, is

$$
\mathbf{S}_{ta} = \mathbf{X} \left[\begin{array}{cc} \exp\{-\gamma(s)\ell_a\} & 0 \\ 0 & \exp\{+\gamma(s)\ell_a\} \end{array} \right] \bar{\mathbf{X}} \tag{3}
$$

where $\mathbf{X}$ is the transmission scattering matrix of the left launcher and

$$
\bar{\mathbf{X}} = \mathbf{P}\mathbf{X}^{-1}\mathbf{P},\ \ \mathbf{P} = \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right] \tag{4}
$$

that means

$$
\mathbf{S}_{ta} = \mathbf{X}\,\mathbf{D}\,(\mathbf{P}\mathbf{X}^{-1}\mathbf{P}) \tag{5}
$$

where $\mathbf{D}$ is the diagonal matrix of the propagation functions. Besides, the Nicolson-Ross method assumes as launcher a pure impedance discontinuity, *i.e.,*

$$
\mathbf{X} = \frac{1}{2Y_o} \left[\begin{array}{cc} (Y_o + Y) & (Y_o - Y) \\ (Y_o - Y) & (Y_o + Y) \end{array} \right] \tag{6}
$$

where Y_o and Y are the characteristic admittance of the measurement system and of the waveguide segment, respectively. This matrix and its inverse are invariant for rows and columns permutations, *i.e.,* $\mathbf{P}\mathbf{X}^{-1}\mathbf{P} = \mathbf{X}^{-1}$. In this case, therefore, the propagation functions are the eigenvalues of $\mathbf{S}_{ta}$ and the Nicolson-Ross method amounts for estimating the waveguide propagation function as the eigenvalues of the transmission scattering matrix of the setup

$$\mathbf{S}_{ta}\mathbf{X} = \mathbf{X}\,\mathbf{D} \tag{7}$$

In contrast, if this symmetry condition does not hold, then $\mathbf{X}$ and $\mathbf{D}$ are related by

$$(\mathbf{S}_{ta}\mathbf{P})\,\mathbf{X} = \mathbf{X}\,(\mathbf{D}\mathbf{P}) \tag{8}$$

and $\mathbf{D}$ cannot be computed from $\mathbf{S}_{ta}$ only.

3 RESULTS

In order to verify the performance of the double-delay method on real measurement problems, we have applied it to the estimation of the permittivity of a reference liquid as methanol. The S-parameters of two coaxial airlines of different lengths (see Fig. 1, Maury Microwave Airline, length $\ell_1 = 10.5\,\mathrm{cm}$ and $\ell_2 = 30\,\mathrm{cm}$, load shield and inner conductor radii 3.5 mm and 1.5 mm, respectively, dc-resistance of inner conductor $9.4\,\mathrm{m}\Omega/\,\mathrm{m}$) filled with methanol have been measured with a Network Analyzer (Agilent E5071B) in the range $100\,\mathrm{MHz}-1\,\mathrm{GHz}$ at temperature of $25^o\mathrm{C}$. The magnitude and phase of the measured S-parameters for the short and long airline are shown in Fig. 2 and Fig. 3 respectively. When the double-delay method, described in

Figure 1: The two coaxial airlines used for the measurements of the S-parameters

section 2, is applied to the scattering parameters and the estimated propagation function is inverted, the complex relative permittivity of the dielectric filling the airlines can be obtained (Fig. 4). This estimation leads to a real part of the relative permittivity value around 32, that coincides with the nominal value of methanol. The complex permittivity obtained with the double-delay method (solid line) has been compared with the dispersion curve of methanol found in the literature (dashed line) [4] as shown in Fig. 4. The dispersion curves of [4] has been obtained by fitting the Cole-Cole equation to fifteen measured data point in the range $100\,\mathrm{MHz} - 70\,\mathrm{GHz}$ for each temperature values. For the real part of the permittivity of the Jordan's curves, the fitting error was estimated to be around 0.5, which is consistent with the permittivity values estimated here by the double delay method.

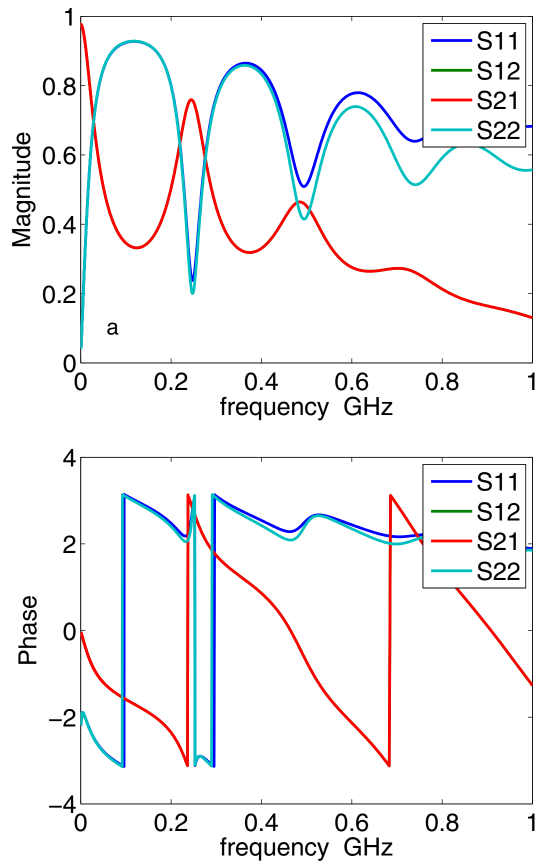

Figure 2: Magnitude (panel a) and phase (panel b) of the measured S-parameters for the short coaxial airline.

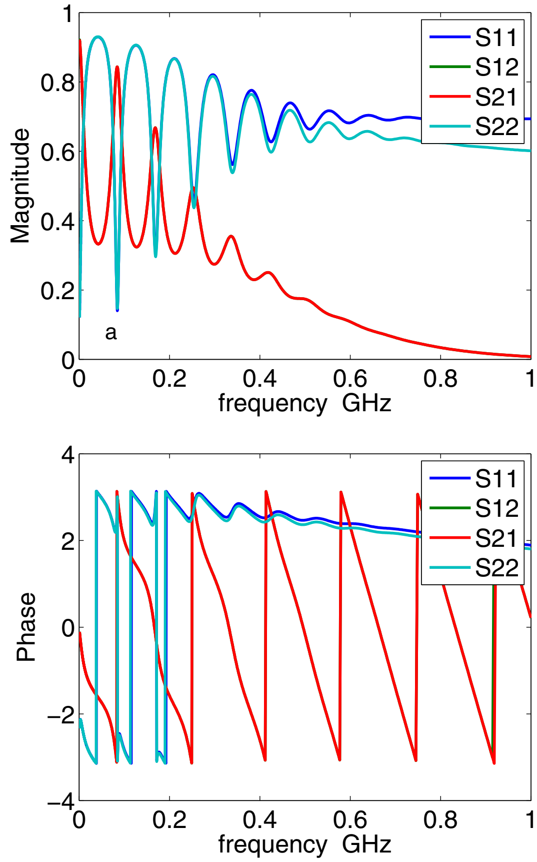

Figure 3: Magnitude (panel a) and phase (panel b) of the measured S-parameters for the long coaxial airline.

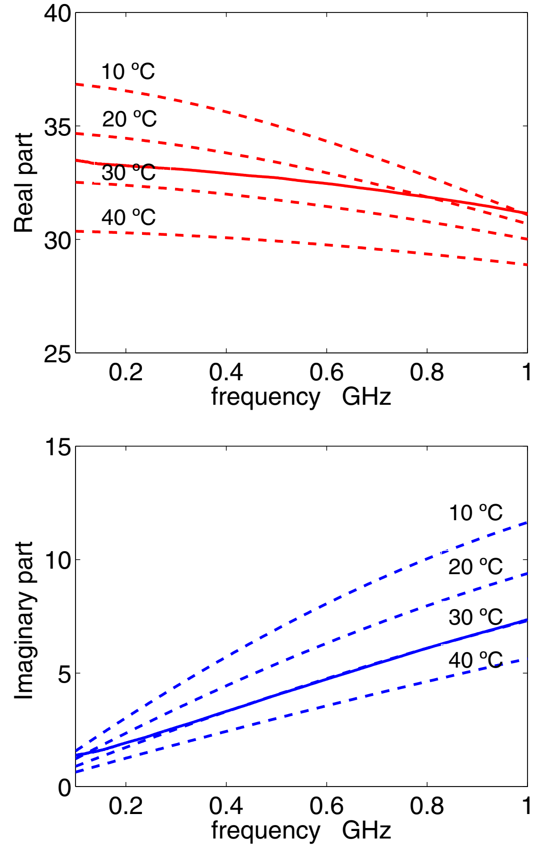

Figure 4: Solid line: complex permittivity of Methanol obtained with the deembedding procedure. Dashed line: dispersion curve of methanol for different room temperature [4].

4 CONCLUSIONS

The estimation of complex permittivity from the scattering responses of setups composed of a uniform waveguide and its launchers has been addressed by means of the the double-delay method.

The obtained results show that this method is able to de-embed the effects of arbitrary launchers, leading to accurate estimation of the complex permittivity over wide frequency ranges.

References

[1] A. M. Nicolson and G. F. Ross. "Measurement of the intrinsic properties of materials by time-domain techniques". *IEEE Trans. on Instr. and Measurement*, IM-19(4):377-402, 1970.

[2] J. C. Rautio. "A de-embedding algorithm for electromagnetics". *International Journal of Microwave and Millimeter-Wave Computer-Aided Engineering*, 1(3):282-2S7, 1991.

[3] M. Farina, A. Morini, and T. Rozzi. "A calibration approach for the segmentation and analysis of microwave circuits". *IEEE Trans. on Microwave Theory and Techniques*, 55(10):2124-2134, 2007.

[4] B.P. Jordan, R.J. Sheppardt and S. Szwarnowskit. "The dielectric properties of formamide, ethanediol and methanol". *J. Phys. D: Appl. Phys.*, 11:695-701, 1978.

[5] J. M. Song, et al., "De-embedding techniques for embedded microstrips". *Microwave and Optical Technology Letters*, 42(1):54, 2004.